Monty Hall problem

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I've always been intrigued by puzzles with a very intuitive answer that's completely and utterly wrong. The earliest of these brain teasers that I remember is this one, the Monty Hall or game show problem. This blog post will be the first in a series in which will focus on the brain teasers I found interesting enough to highlight here. The solution will of course be included (click below the problem to reveal the spoiler). However I'd recommend first trying to solve it (unless you already know the answer of course).

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Click for solution

A player who stays with the initial choice wins in only one out of three of these equally likely possibilities, while a player who switches wins in two out of three.
An intuitive explanation is that if the contestant picks a goat (2 of 3 doors) the contestant will win the car by switching as the other goat can no longer be picked, while if the contestant picks the car (1 of 3 doors) the contestant will not win the car by switching. The fact that the host subsequently reveals a goat in one of the unchosen doors changes nothing about the initial probability.

Another way to understand the solution is to consider the two original unchosen doors together. "Monty is saying in effect: you can keep your one door or you can have the other two doors". The 2/3 chance of finding the car has not been changed by the opening of one of these doors because Monty, knowing the location of the car, is certain to reveal a goat. So the player's choice after the host opens a door is no different than if the host offered the player the option to switch from their original chosen door to the set of both remaining doors. The switch in this case clearly gives the player a 2/3 probability of choosing the car.

In other words: "By opening his door, Monty is saying to the contestant 'There are two doors you did not choose, and the probability that the prize is behind one of

them is 2/3. I'll help you by using my knowledge of where the prize is to open one of those two doors to show you that it does not hide the prize. You can now take advantage of this additional information. Your choice of door A has a chance of 1 in 3 of being the winner. I have not changed that. But by eliminating door C, I have shown you that the probability that door B hides the prize is 2 in 3.'"

The solution will probably be more intuitive with 1,000,000 doors rather than 3. In this case there are 999,999 doors with goats behind them and one door with a prize. After the player picks a door the host opens all but 1 of the remaining doors. On average, in 999,999 times out of 1,000,000, the remaining door will contain the prize. Intuitively, the player should ask how likely is it, that given a million doors, he or she managed to pick the right one initially.

This problem (and the above solution) were taken from: https://en.wikipedia.org/wiki/Monty_Hall_problem. Check it out for more information about this problem and similar one's.